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Hawking radiation of nonsingular black holes in two dimensions

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Abstract: In this letter we study the process of Hawking radiation of a black hole assuming the existence of a limiting physical curvature scale. The particular model is constructed using the Limiting Curvature Hypothesis (LCH) and in the context of two-dimensional dilaton gravity. The black hole solution exhibits properties of the standard Schwarzschild solution at large values of the radial coordinate. However, near the center, the black hole is nonsingular and the metric becomes that of de Sitter spacetime. The Hawking temperature is calculated using the method of complex paths. We find that such black holes radiate eternally and never completely evaporate. The final state is an eternally radiating relic, near the fundamental scale, which should make a viable dark matter candidate. We briefly comment on the black hole information loss problem and the production of such black holes in collider experiments.

Keywords: Black Holes in String Theory, 2D Gravity, Black Holes.

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Contents

1. Introduction

Two of the outstanding questions in physics today are how should Einstein's theory of gravity be modified in regions of spacetime with high curvatures, and what is the final state of an evaporating black hole. The physics required to answer both of these questions rests in an as of yet unknown theory of quantum gravity.

In general, theories which report to describe features of quantum gravity (including some string theories) contain higher derivative corrections to the Einstein-Hilbert action [1]–[6]. For example, perturbative string theory predicts an infinite series of correction terms to the Einstein equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + \mathcal{O}(\alpha' R^2) + \mathcal{O}(\alpha'^2 R^4) + \cdots = 0\,. \tag{1.1}$$

These correction terms become important at energies near the string scale $\sqrt{\alpha'} = M_s^{-1}$. It is possible that such terms will drastically affect the structure of singularities in both cosmological and black hole spacetimes. Perhaps quantum gravity will some how remove the singularities, leading to geodesically complete spacetimes.

Two examples of higher-derivative gravitational theories which eliminate the singularity in the Pre-Big-Bang model (allowing for a successful graceful exit) are constructed in [7, 8]. These models utilize the Limiting Curvature Hypothesis (LCH) of [9]. The goal of the LCH is to capture features of quantum gravity in an effective theory which interpolates between General Relativity at low energies and a nonsingular spacetime at high curvatures. The LCH has been studied in a variety of black hole and cosmological spacetimes [7]– [14] (also see [15]– [17]). The assumption of the existence of a limiting curvature is well motivated given the existence of a fundamental length scale ℓ_f (such as the Planck length ℓ_{pl}

or the string length ℓ_s). If there is a fundamental length then it follows (from dimensional arguments) that all curvature invariants are bounded

$$
\begin{aligned}
&|R| < \ell_f^{-2}\,, \qquad |R_{\alpha\beta}R^{\alpha\beta}| < \ell_f^{-4}\,, \\
&|C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}| < \ell_f^{-8}\,, \ldots
\end{aligned}
\tag{1.2}
$$

In this letter we investigate the process of Hawking radiation of a nonsingular black hole in $(1+1)$-dimensional dilaton gravity [11]. The construction involves concepts from the LCH. However, since the Ricci scalar is the only invariant in two dimensions, the role of the higher derivative correction terms is facilitated by the dilaton and its potential. The resulting solution is nonsingular everywhere and resembles the Schwarzschild black hole at large values of the radial coordinate. At $r = 0$ the solution approaches the de Sitter metric. Cosmologically desirable consequences may arise if the universe is generated at the interior of a black hole (see, e.g. [18] and the references therein).

Although it is natural to question the physical validity of results in $2D$ dilaton gravity, there are strong motivations for using the theory as a toy model of quantum gravity and to study the process of black hole evaporation. In particular, a certain limit of superstring theory yields a generalized dilaton theory as the effective action (see e.g. [28]). This simplified model allows for completely analytic calculations, possibly capturing non-perturbative features of the physics, evaporation and final state of realistic four-dimensional black holes.[1] Further motivation is provided by [29], where it is argued that the thermodynamics of black hole in $D \geq 4$ is effectively described by the thermodynamics of black holes in two-dimensional dilaton gravity theories. Specifically, it is shown that the Bekenstein-Hawking (BH) entropies of single-charged dilaton black holes and dilaton p-branes with an arbitrary dilaton coupling parameter in arbitrary spacetime dimensions are exactly reproduced by the BH entropy of the two-dimensional black hole in the associated $2D$ dilaton gravity model. We are also inspired by the successful generalization of a nonsingular cosmological model in two dimensions [13] to a nonsingular four-dimensional model [7, 8] using dilaton gravity and the limiting curvature construction mentioned above. Based on this encouraging evidence, we believe that the two-dimensional results presented in this paper should apply to more realistic four-dimensional models.

2. A nonsingular black hole

Here we reconstruct the nonsingular two-dimensional dilaton black hole solution presented in [11]. We begin with the most general lagrangian for gravity with a scalar field (e.g. , dilaton gravity) in $(1+1)$-dimensions [30]:

$$
\mathcal{L} = \sqrt{-g}\left(D(\varphi)\,R + G(\varphi)\,(\nabla\varphi)^2 + H(\varphi)\right)\,. \tag{2.1}
$$

By performing a conformal transformation

$$
g_{\alpha\beta} \to e^{2\sigma(\varphi)} g_{\alpha\beta}\,, \tag{2.2}
$$

[1] For some early references to two-dimensional gravity see [19]– [27].

and requiring that

$$4\frac{d\sigma}{d\varphi}\frac{dD}{d\varphi} = -G(\varphi)\,, \tag{2.3}$$

it is possible to conformally transform away the kinetic term for φ and we may simply start with the action

$$S = \int d^2x\sqrt{-g}\,(D(\varphi)\,R + V(\varphi))\,, \tag{2.4}$$

where $V(\varphi) = e^{2\sigma(\varphi)}H(\varphi)$.[2]

Varying this action with respect to φ and the metric tensor yields

$$\begin{aligned} -\frac{\partial V(\varphi)}{\partial\varphi} &= \frac{\partial D(\varphi)}{\partial\varphi}R && (2.5)\\ V(\varphi)\,g_{\alpha\beta} &= 2(\nabla^2 g_{\alpha\beta} - \nabla_\alpha\nabla_\beta)D(\varphi)\,, && (2.6)\end{aligned}$$

respectively. We may redefine φ so that $D(\varphi) = 1/\varphi$ and assume a spherically symmetric and static metric

$$ds^2 = -n(r)dt^2 + p(r)dr^2\,. \tag{2.7}$$

In the "Schwarzschild gauge" $p(r) = 1/n(r)$ and the EOM become

$$\begin{aligned} \varphi^3 V(\varphi) - 4n\varphi'^2 + n'\varphi'\varphi + 2n\varphi\varphi'' &= 0\\ \frac{\partial V}{\partial\varphi} + \varphi^{-2}n'' &= 0\\ V(\varphi) + \varphi^{-2}\varphi' n' &= 0\,. \end{aligned} \tag{2.8}$$

Implementing the limiting curvature hypothesis is equivalent to identifying a class of potentials $V(\varphi)$ so that R is bounded for all values of the radial coordinate from 0 to ∞. The form of the potential is restricted by considering the behavior of the system at low and high curvatures. We require $n(r)$ to reduce to the Schwarzschild solution in regions of spacetime with small curvature. Mathematically,

$$n(r) \to \left(1 - \frac{2m}{r}\right) \qquad \text{when} \qquad r\to\infty,\ \varphi\to 0\,. \tag{2.9}$$

From equation (2.5) it is easy to see that limiting $V'(\varphi)$ in the high curvature regime (i.e. $r\to 0$, $\varphi\to\infty$) will result in the limitation of R.

While there are many potentials that interpolate correctly between the above asymptotic regimes, a simple potential which satisfies the above criteria is

$$V(\varphi) = \frac{2mA^3\varphi^2}{1 + mA^3 l^2\varphi^3}\,, \tag{2.10}$$

[2] Note that in [31], it is argued that under certain circumstances conformally related theories may not be equivalent.

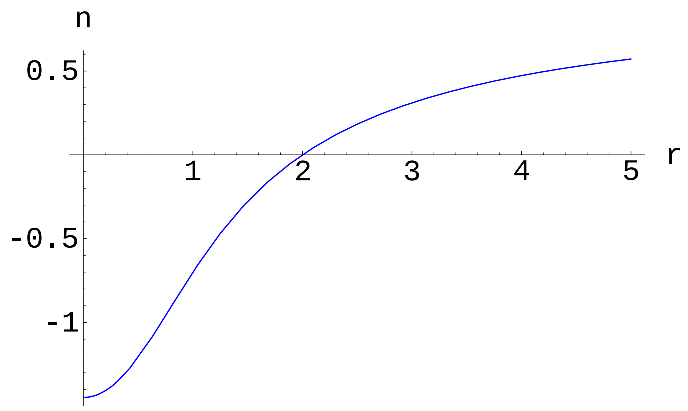

Figure 1: A plot of $n(r)$ for the values $l = m = 1$. (These values are used in the plots throughout the paper.) Far from the point $r = 0$ the solution is indistinguishable from the Schwarzschild solution. The function n vanishes at the horizon $r = r_0 = 2m$. Unlike the Schwarzschild solution, n remains finite at $r = 0$.

where A is a constant and l is a constant representing the limiting fundamental scale.[3] Using this potential in the EOM one finds the exact solutions

$$\begin{aligned}
\varphi(r) &= \frac{1}{Ar}\,, \\
n(r) &= \frac{1}{3}\left(\frac{m}{l}\right)^{2/3} \ln\left\{\frac{r^2-(ml^2)^{1/3}r+(ml^2)^{2/3}}{r_0^2-(ml^2)^{1/3}r_0+(ml^2)^{2/3}}\left(\frac{r_0+(ml^2)^{1/3}}{r+(ml^2)^{1/3}}\right)^2\right\} + \\
&\quad + \frac{2}{\sqrt{3}}\left(\frac{m}{l}\right)^{2/3}\left\{\arctan\left(\frac{2r-(ml^2)^{1/3}}{\sqrt{3}(ml^2)^{1/3}}\right) - \arctan\left(\frac{2r_0-(ml^2)^{1/3}}{\sqrt{3}(ml^2)^{1/3}}\right)\right\}, \qquad (2.11)
\end{aligned}$$

where r_0 is the location of the black hole horizon. The function $n(r)$ is plotted below in figure 1. When the mass of the black hole is large, n reduces to $(1 - 2m/r)$ as expected.

The Ricci scalar R (the only curvature invariant in two-dimensions) remains finite for all positive value of the radial coordinate r,

$$R = \frac{2m(2r^3 - l^2 m)}{(r^3 + l^2 m)^2}\,, \qquad (2.12)$$

hence, the spacetime is singularity free (see figure 2). Note that R changes sign at the value $r = (l^2 m/2)^{2/3}$ and takes on its limiting value $R_{\max}$ at $r = 0$,

$$R_{\max} = -\frac{2}{l^2}\,. \qquad (2.13)$$

[3]Note that this potential is only one possible potential that will force the curvature to be limited. While it is not a unique choice all potentials that satisfy the asymptotic conditions, limiting the curvature, will give the same result. For a more detailed explanation of the construction of $V(\varphi)$ we refer the reader to the original source [11].

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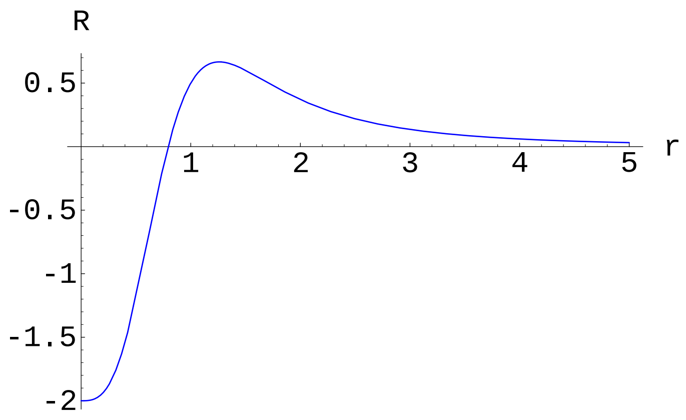

Figure 2: The curvature R remains finite for all r from $r = 0$ to $r = \infty$.

Near $r = 0$ the spacetime becomes de Sitter.[4] To see this, expand n around the point $r = 0$:

$$n(r) \simeq -C + \frac{r^2}{l^2} + \mathcal{O}[(r)^3] \,. \tag{2.14}$$

Ignoring terms of order r^3 and higher, the line element becomes

$$ds^2 = -\left(\frac{r^2}{l^2} - C\right) dt^2 + \left(\frac{r^2}{l^2} - C\right)^{-1} dr^2 \,, \tag{2.15}$$

where C is a constant. Taking $C = 1$, we may define a new coordinate τ given by

$$\frac{d\tau}{dr} = \left(\left(\frac{r}{l}\right)^2 - 1\right)^{-1/2} \,, \tag{2.16}$$

which gives

$$\frac{\tau(r)}{l} = \operatorname{arccosh}\left(\frac{r}{l}\right) \,. \tag{2.17}$$

In the (t, τ) coordinates the metric takes on the recognizable de Sitter form

$$ds^2 = d\tau^2 - \sinh^2\left(\frac{\tau}{l}\right) dt^2 \,. \tag{2.18}$$

Another conceivably interesting region is near the fundamental scale $r \simeq l$. Expanding around $r = l$, n takes the form

$$n(r) \simeq C' + \frac{2m}{l(l+1)}(r - l) + \mathcal{O}[(r - l)^2] \,. \tag{2.19}$$

Here we choose a new coordinate χ (and $C' = 1$) defined by

$$\frac{d\chi}{dr} = \left(\frac{2m(r - l)}{l(l+1)} - 1\right)^{-1/2} \,. \tag{2.20}$$

[4]In the conventions of [11], the cosmological constant ($\Lambda \propto R$) for de Sitter space is negative.

In the (t, χ) coordinates the metric becomes

$$ds^2 = d\chi^2 - \alpha^2 \chi^2 dt^2 \,, \tag{2.21}$$

where the constant $\alpha = m/l(l+1)$. This is simply the Milne metric.

One final comment is in order concerning the above solution. In [32], Myers and Horowitz argue that spacetime singularities should play a useful role in gravitational theories by eliminating unphysical solutions. Furthermore, they suggest that any modified theory of gravity must have singularities in order to posses a stable ground state. The argument goes as follows: even if the theory claims to have a ground state with $E < 0$, one can always start with the Schwarzschild metric with $m < E$ and argue that it must be singular. Removing all singularities leads to the existence of states with arbitrarily negative energy. They suggest that a new mechanism must be found to prevent a state which resembles the negative mass Schwarzschild solution from existing in the theory.

In the above model, we may argue that the negative mass Schwarzschild solution is not a well behaved solution, in the sense that a singularity is present for $m < 0$ (see equation (2.12)). Furthermore, we may argue that the negative energy solutions may simply be thrown out due to their pathological, singular nature. Negative mass Schwarzschild solutions are not possible asymptotic solutions of the theory since a singularity is present at finite r. Thus, the instability of the theory for $m < 0$ discussed in [32] does not occur in our theory.[5]

3. Hawking radiation and the method of complex paths

We now examine the process of Hawking radiation of the nonsingular black hole solution obtained in the previous section. One of the simplest ways to do this is to employ the method of complex path analysis introduced by Landau [33] and applied to Schwarzschild-like spacetimes by Srinivasan and Padmanabhan [34] (also see [35], for a related treatment). Here we use the method to calculate the Hawking temperature of a two-dimensional, static and spherically symmetric metric with horizon. We will then apply this formula to the black hole solution derived above.

The line element is given by equation (2.7) in the Schwarzschild gauge $p(r) = 1/n(r)$:

$$ds^2 = -n(r)dt^2 + n^{-1}(r)dr^2 \,. \tag{3.1}$$

We assume that $n(r)$ vanishes at some r_0 and $n'(r)$ is finite and nonzero at r_0. This indicates that there is a horizon at the value $r = r_0$. One must of course check that the singularity at r_0 is only a mathematical one. In our case this is simple since the only curvature invariant in two-dimensions is R, which remains finite at r_0 (see equation (2.12)). Expanding $n(r)$ around the point r_0 gives

$$n(r) = n'(r_0)(r - r_0) + \mathcal{O}[(r - r_0)^2] \equiv \mathcal{R}(r_0)(r - r_0) \,, \tag{3.2}$$

where we assume that $\mathcal{R}(r_0) \neq 0$.

[5] I would like to thank R. Myers for useful discussions concerning this point.

Now consider a scalar field which satisfies the Klein-Gordon equation

$$\left(\Box - \frac{m_0^2}{\hbar^2}\right)\Phi = 0\,. \tag{3.3}$$

If this field is propagating in the background spacetime given by equation (3.1), then

$$-\frac{1}{n(r)}\frac{\partial^2\Phi}{\partial t^2} + \frac{\partial}{\partial r}\left(n(r)\frac{\partial\Phi}{\partial r}\right) = \frac{m_0^2}{\hbar^2}\Phi\,. \tag{3.4}$$

The semiclassical wavefunctions satisfying the above are obtained by making the standard ansatz

$$\Phi(r,t) = e^{\{\frac{i}{\hbar}\mathcal{S}(r,t)\}}\,, \tag{3.5}$$

where $\mathcal{S}$ is a functional which will be expanded in powers of $\hbar$. Substituting this ansatz into equation (3.4) gives,

$$-\frac{1}{n(r)}\left(\frac{\partial\mathcal{S}}{\partial t}\right)^2 + n(r)\left(\frac{\partial\mathcal{S}}{\partial r}\right)^2 + m_0^2 - \frac{i}{\hbar}\left[\frac{1}{n(r)}\frac{\partial^2\mathcal{S}}{\partial t^2} - n(r)\frac{\partial^2\mathcal{S}}{\partial r^2} - \frac{dn(r)}{dr}\frac{\partial\mathcal{S}}{\partial r}\right] = 0\,. \tag{3.6}$$

We now expand $\mathcal{S}$ in a power series of $\hbar/i$:

$$\mathcal{S}(r,t) = \mathcal{S}_0(r,t) + \frac{\hbar}{i}\mathcal{S}_1(r,t) + \left(\frac{\hbar}{i}\right)^2\mathcal{S}_2(r,t) + \cdots\,, \tag{3.7}$$

and substitute the result into equation (3.6). Neglecting terms of order $\hbar/i$ or higher we have

$$-\frac{1}{n(r)}\left(\frac{\partial\mathcal{S}_0}{\partial t}\right)^2 + n(r)\left(\frac{\partial\mathcal{S}_0}{\partial r}\right)^2 + m_0^2 = 0\,. \tag{3.8}$$

This is simply the Hamilton-Jacobi equation satisfied by a particle of mass m_0 moving in a background spacetime with metric (3.1). The solution to this equation is

$$\mathcal{S}_0(r,t) = -\mathcal{E}t \pm \int^r \frac{dr}{n(r)}\sqrt{\mathcal{E}^2 - m_0^2 n(r)}\,, \tag{3.9}$$

where $\mathcal{E}$ is a constant and is identified with the energy. For simplicity we take $m_0 = 0$. [6] Now it is possible to solve equation (3.8) exactly.

Using the usual saddle point method, the semiclassical propagator $K(x'', x')$ for a particle propagating from a spacetime point $x'' = (t_1, r_1)$ to a point $x' = (t_2, r_2)$ is

$$K(x'', x') = \mathcal{N} e^{\frac{i}{\hbar}\mathcal{S}_0(x'',x')}\,, \tag{3.10}$$

where $\mathcal{S}_0$ is the action functional satisfying the classical Hamilton-Jacobi equation in the massless limit and $\mathcal{N}$ is a suitable normalization constant. With the solution to equation (3.8):

$$\mathcal{S}_0(x'', x') = -\mathcal{E}(t_2 - t_1) \pm \mathcal{E}\int_{r_1}^{r_2}\frac{dr}{n(r)}\,, \tag{3.11}$$

[6] Note that the results do not change for $m_0 \neq 0$ [34].

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we can calculate the amplitudes and probabilities of emission and absorption through the event horizon at r_0. Note that the sign ambiguity in equation (3.11) corresponds to the outgoing ($\partial\mathcal{S}_0 > 0$) or ingoing ($\partial\mathcal{S}_0 < 0$) massless particles.

If the points x'' and x' are on opposite sides of the event horizon then the above integral diverges (since n^{-1} diverges at $r = r_0$). Therefore, to evaluate the integral we employ the calculus of residues and choose the contour over which the integral is to be performed around the point $r = r_0$.

Consider an outgoing particle at $r = r_1 < r_0$. The modulus squared of the amplitude for this particle to cross the horizon gives the probability of emission of the particle. Invoking the usual "$i\epsilon$" prescription, the contribution to $\mathcal{S}_0$ in the ranges $(r, r_0 - \epsilon)$ and $(r_0 + \epsilon, r_2)$ is real. We take the contour to lie in the upper complex plane and find

$$\begin{aligned} \mathcal{S}_0[\text{emission}] &= -\mathcal{E} \lim_{\epsilon\to 0} \int_{r_0-\epsilon}^{r_0+\epsilon} \frac{dr}{n(r)} + \text{real part} \\ &= \frac{i\pi\mathcal{E}}{\mathcal{R}(r_0)} + \text{real part}\,, \end{aligned} \tag{3.12}$$

where the minus sign in front of the integral corresponds to the initial condition that $(\partial\mathcal{S}_0/\partial r) > 0$ at $r = r_1 < r_0$, and $\mathcal{R}(r_0)$ is given by equation (3.2).

Now consider an ingoing particle with $(\partial\mathcal{S}_0/\partial r) > 0$ at $r = r_2 > r_0$. The modulus squared of the amplitude for this particle to cross the horizon gives the probability of absorption of the particle into the horizon. Choosing the contour to lie in the upper complex plane gives

$$\begin{aligned} \mathcal{S}_0[\text{absorption}] &= -\mathcal{E} \lim_{\epsilon\to 0} \int_{r_0+\epsilon}^{r_0-\epsilon} \frac{dr}{n(r)} + \text{real part} \\ &= -\frac{i\pi\mathcal{E}}{\mathcal{R}(r_0)} + \text{real part}\,. \end{aligned} \tag{3.13}$$

This result agrees with the calculation of an outgoing particle $(\partial\mathcal{S}_0/\partial r) > 0$ at $r = r_2 > r_0$. Here the contour is taken in the lower half-plane and the amplitude for the particle to cross the horizon is the same as that of ingoing particle due to time reversal invariance.

Squaring the modulus to get the probability gives,

$$\begin{aligned} P[\text{emission}] &\propto e^{-2\pi\mathcal{E}/\hbar\mathcal{R}}\,, \\ P[\text{absorption}] &\propto e^{2\pi\mathcal{E}/\hbar\mathcal{R}}\,, \end{aligned} \tag{3.14}$$

implying that

$$P[\text{emission}] = e^{-4\pi\mathcal{E}/\hbar\mathcal{R}} P[\text{absorption}]\,. \tag{3.15}$$

Comparing this formula with the relation due to Hawking and Hartle [36]:

$$P[\text{emission}] = e^{(-\beta\mathcal{E})} P[\text{absorption}]\,, \tag{3.16}$$

we find the identification

$$\beta^{-1} = \frac{\hbar|\mathcal{R}|}{4\pi}\,. \tag{3.17}$$

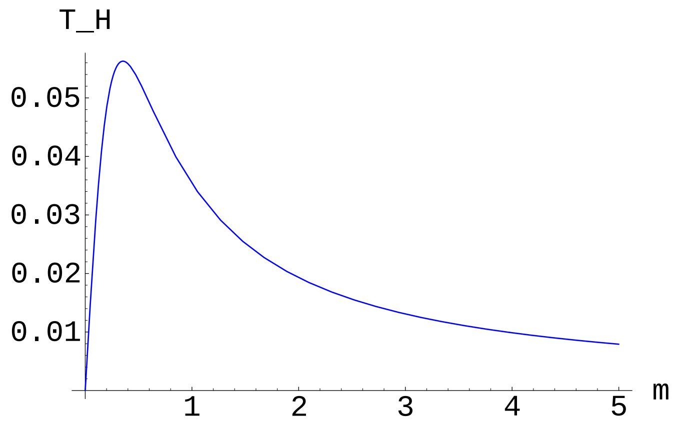

Figure 3: The Hawking temperature T_H asymptotically goes to zero as the mass goes to zero.

The method described above was shown to correctly reproduce the Hawking temperatures of the Schwarzschild black hole

$$\beta^{-1} = \frac{\hbar}{8\pi m}\,, \tag{3.18}$$

de Sitter spacetime and Rindler spacetime [34], and is valid for a variety of coordinate systems [37]. The method employs simple quantum mechanics and reproduces results obtained by much more difficult means such as the calculation of Bogoliubov coefficients and other traditional techniques, e.g. [36, 38, 39].

4. An eternally evaporating black hole

Calculating the Hawking temperature of the black hole (given by the metric (3.1), with equation (2.11)) is now a simple task. Expanding $n(r)$ near r_0 gives

$$n(r) = \frac{2mr_0}{l^2m + r_0^3}(r - r_0) + \mathcal{O}[(r - r_0)^2]\,. \tag{4.1}$$

In order to agree with the Schwarzschild solution we take $r_0 = 2m$, from which $\mathcal{R}(2m) = 4m/(l^2 + 8m^2)$. Hence, the Hawking temperature of the black hole is given by (see equation (3.17)),

$$T_H = \frac{\hbar m}{\pi(l^2 + 8m^2)}\,. \tag{4.2}$$

This temperature is plotted in figure 3.

The most important feature of this solution is that $T_H \to 0$ as $m \to 0$, in contrast to the standard Schwarzschild black hole which approaches infinite temperature in the zero mass limit (see equation (3.18)). Notice that in the limit of large m (or in the limit as the fundamental parameter $l \to 0$) the formula for T_H appropriately reduces to the Schwarzschild temperature. At the maximum temperature $T_{max} = \hbar/4\sqrt{2}\pi l$, the curvature at the horizon, remains less than the magnitude of the limiting curvature. Hence, we are comforted that our semiclassical treatment of the radiation should remain a good approximation.

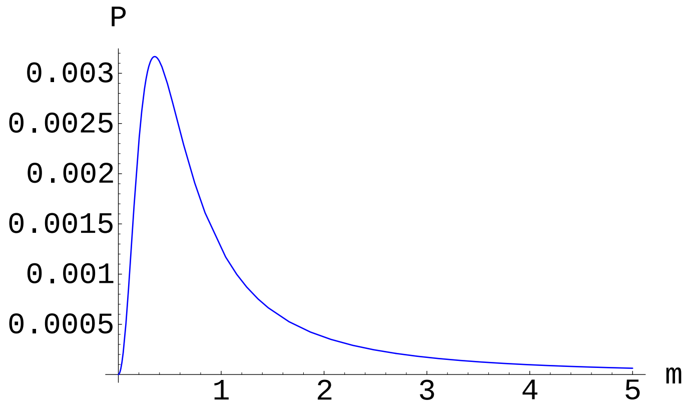

Figure 4: The power $\mathcal{P} \sim -dm/dt$ versus mass m.

A two dimensional version of Stefan's law gives the total power radiated by the black hole:

$$\mathcal{P} \sim -\frac{dm}{dt} \sim T^2 \,. \tag{4.3}$$

The power is plotted in figure 4.

Using the power it is possible to estimate the evaporation time of the black hole

$$t_{\rm evap} \sim \frac{m}{|dm/dt|} \sim \frac{(l^2 + 8m^2)^2}{m} \,, \tag{4.4}$$

which is infinite in the limit $m \to 0$. In the limit of large mass (or taking $l \to 0$) the evaporation time reduces to the familiar formula for the Schwarzschild black hole $t_{evap} \sim m^3$. Our result implies that the black hole will radiate eternally. The mass will decrease as in the Schwarzschild case until most of the mass is radiated away, at which point the radiation decreases and one is left with a very slowly radiating, small black hole.[7] (At the maximum temperature the black hole has a Schwarzschild radius of $r_s = l/\sqrt{2}$.)

Note that if this result applies to realistic black holes, then it should significantly affect the analysis of evaporating black holes created at CERN's Large Hadron Collider (LHC) or in future collider experiments (such as CLIC and VLHC). Most current discussions of black holes created in the lab are based on semi-classical calculations that are valid only when the mass of the black hole is much larger than M_{pl}. When the black hole mass approaches the fundamental scale the physics required to understand the process of Hawking radiation is rooted in a theory of quantum gravity. Our classical intuition concerning the creation of black holes in the lab may require refinement. (This is indicated by the above result). Further observational consequences of black hole remnants will be discussed in a future work [41].

[7] Note that a similar behavior was found in [40]. Here the authors considered a microcanonical treatment of black holes.

5. Miniature black holes as dark matter candidates

One of the greatest challenges of cosmology is to determine the nature of the dark matter/energy which makes up most of the matter in the Universe. The eternal black holes we have described above could have very significant consequences for cosmology, since they have the potential to contribute a sizeable amount of the dark matter in the Universe today (black hole remnants could behave as weakly interacting matter particles (WIMPS) [42]).[8] It is possible that in the early Universe geometric fluctuations produced a thermal (Boltzmann) distribution of black holes (see e.g.,[46]). During the expansion of the Universe these black holes would slowly decay into radiation and very small black holes near or below the fundamental scale which would still be present today.

6. Conclusions

In this paper we have explored the process of Hawking radiation within the context of a specific model for a nonsingular black hole. We find that the black hole radiation mimics the Schwarzschild case for large values of the mass, but then reaches a maximum (determined by the fundamental scale) and then slowly decreases for all values of the time coordinate. The resulting miniature black holes could play an important role as dark matter candidates.

A brief comment on the black hole information loss problem is warranted. The above model does not suffer from the traditional black hole information loss problem. All the information which falls into the black hole is transported into the nonsingular black hole interior. In principle, an observer travelling into the black hole should be able to recover this information. Because the black hole never completely evaporates, the information is never truly lost.

A standard argument against the existence of stable black hole remnants is that it seems physically unreasonable for a large amount of information to be carried within a small (planckian) volume [47]. One possible way to circumvent this issue was proposed within the context of charged dilatonic black holes [48]–[50]. Here it is suggested that what appears to be a small volume to an observer outside of the black hole is actually an infinite volume tube capable of storing infinite information. The scenario discussed in this paper shares this feature. The black hole is viewed from the outside as being a small remnant. The large interior core contains the missing information which may be accessed only by travelling into the black hole. In [18] it was argued that the size of the universe inside the black hole is infinite.[9]

Of course, it remains to be shown that such physical properties will carry over to realistic four-dimensional black holes. This possibility is currently being investigated [51]. In order to generalize the model to four dimensions we must use a higher derivative theory of

[8] For some early references on black holes as dark matter candidates see [43]–[45].

[9] One problem which may remain in this scenario is that a large number of fundamental-mass particles will appear in loops and in the thermodynamic ensemble leading to a large degeneracy. It is conceivable, however, that they could be cured by suppressed amplitudes for creating such large cores if these amplitudes decrease sufficiently rapidly with the information content [47].

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gravity. The gravitational action should admit a solution which resembles the Schwarzschild black hole at large distances but with the singularity replaced by a de-Sitter universe. The method of construction will be similar to that in [8].

Our main conclusion is as follows. A specific construction, based on the notion of a limiting curvature, is capable of removing singularities in cosmological models. In this paper we have argued that this construction predicts long-lived black hole relics with observational consequences and possibly solves the information loss problem.

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References

[1] G. 't Hooft and M.J. Veltman, *One loop divergencies in the theory of gravitation*, *Ann. Poincaré Phys. Theor.* **A 20** (1974) 69.

[2] J. Scherk and J.H. Schwarz, *Dual models and the geometry of space-time*, *Phys. Lett.* **B 52** (1974) 347.

[3] T. Yoneya, *Connection of dual models to electrodynamics and gravidynamics*, *Prog. Theor. Phys.* **51** (1974) 1907.

[4] N.D. Birrell and P.C. Davies, *Quantum fields in curved space*, Cambridge University Press, Cambridge 1982

[5] C. Lovelace, *Strings in curved space*, *Phys. Lett.* **B 135** (1984) 75.

[6] E.S. Fradkin and A.A. Tseytlin, *Quantum string theory effective action*, *Nucl. Phys.* **B 261** (1985) 1.

[7] R.H. Brandenberger, R. Easther and J. Maia, *Nonsingular dilaton cosmology*, *J. High Energy Phys.* **08** (1998) 007 [gr-qc/9806111].

[8] D.A. Easson and R.H. Brandenberger, *Nonsingular dilaton cosmology in the string frame*, *J. High Energy Phys.* **09** (1999) 003 [hep-th/9905175].

[9] M.A. Markov, *Zh. Eksp. Teor. Fiz.* **36** (1982) 214;
Zh. Eksp. Teor. Fiz. **46** (1987) 342.

[10] V. Mukhanov and R.H. Brandenberger, *A nonsingular universe*, *Phys. Rev. Lett.* **68** (1992) 1969.

[11] M. Trodden, V.F. Mukhanov and R.H. Brandenberger, *A nonsingular two-dimensional black hole*, *Phys. Lett.* **B 316** (1993) 483 [hep-th/9305111].

[12] R.H. Brandenberger, V. Mukhanov and A. Sornborger, *A cosmological theory without singularities*, *Phys. Rev.* **D 48** (1993) 1629 [gr-qc/9303001].

[13] R. Moessner and M. Trodden, *Singularity-free two-dimensional cosmologies*, *Phys. Rev.* **D 51** (1995) 2801 [gr-qc/9405004].

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[14] R.H. Brandenberger, *Nonsingular cosmology and Planck scale physics*, gr-qc/9503001.

[15] V.P. Frolov, M.A. Markov and V.F. Mukhanov, *Through a black hole into a new universe?*, *Phys. Lett.* **B 216** (1989) 272.

[16] V.P. Frolov, M.A. Markov and V.F. Mukhanov, *Black holes as possible sources of closed and semiclosed worlds*, *Phys. Rev.* **D 41** (1990) 383.

[17] D. Morgan, *Black holes in cutoff gravity*, *Phys. Rev.* **D 43** (1991) 3144.

[18] D.A. Easson and R.H. Brandenberger, *Universe generation from black hole interiors*, *J. High Energy Phys.* **06** (2001) 024 [hep-th/0103019].

[19] E. D'Hoker and R. Jackiw, *Liouville field theory*, *Phys. Rev.* **D 26** (1982) 3517.

[20] E. D'Hoker and R. Jackiw, *Space translation breaking and compactification in the liouville theory*, *Phys. Rev. Lett.* **50** (1983) 1719.

[21] E. D'Hoker, D.Z. Freedman and R. Jackiw, SO(2, 1) *invariant quantization of the Liouville theory*, *Phys. Rev.* **D 28** (1983) 2583.

[22] C. Teitelboim, *Gravitation and hamiltonian structure in two space-time dimensions*, *Phys. Lett.* **B 126** (1983) 41.

[23] R. Jackiw, *Lower dimensional gravity*, *Nucl. Phys.* **B 252** (1985) 343.

[24] M.O. Katanaev and I.V. Volovich, *String model with dynamical geometry and torsion*, *Phys. Lett.* **B 175** (1986) 413 [hep-th/0209014].

[25] R.B. Mann, A. Shiekh and L. Tarasov, *Classical and quantum properties of two-dimensional black holes*, *Nucl. Phys.* **B 341** (1990) 134.

[26] R.B. Mann, S.M. Morsink, A.E. Sikkema and T.G. Stelle, *Semiclassical gravity in (1+1)-dimensions*, *Phys. Rev.* **D 43** (1991) 3948.

[27] V.A. Kazakov and A.A. Tseytlin, *On free energy of 2-d black hole in bosonic string theory*, *J. High Energy Phys.* **06** (2001) 021 [hep-th/0104138].

[28] D. Grumiller, W. Kummer and D.V. Vassilevich, *Dilaton gravity in two dimensions*, *Phys. Rept.* **369** (2002) 327 [hep-th/0204253].

[29] D. Youm, *Black hole thermodynamics and two-dimensional dilaton gravity theory*, *Phys. Rev.* **D 61** (2000) 044013 [hep-th/9910244].

[30] T. Banks and M. O'Loughlin, *Nonsingular lagrangians for two-dimensional black holes*, *Phys. Rev.* **D 48** (1993) 698 [hep-th/9212136].

[31] D. Grumiller, D. Hofmann and W. Kummer, *Two-dilaton theories in two dimensions*, *Ann. Phys. (NY)* **290** (2001) 69 [gr-qc/0005098].

[32] G.T. Horowitz and R.C. Myers, *The value of singularities*, *Gen. Rel. Grav.* **27** (1995) 915 [gr-qc/9503062].

[33] R.H. Landau, *Quantum mechanics: non-relativistic theory*, Pergamon Press, New York 1977.

[34] K. Srinivasan and T. Padmanabhan, *Particle production and complex path analysis*, *Phys. Rev.* **D 60** (1999) 024007 [gr-qc/9812028].

[35] M.K. Parikh and F. Wilczek, *Hawking radiation as tunneling*, *Phys. Rev. Lett.* **85** (2000) 5042 [hep-th/9907001].

[36] J.B. Hartle and S.W. Hawking, *Path integral derivation of black hole radiance*, *Phys. Rev.* **D 13** (1976) 2188.

[37] S. Shankaranarayanan, T. Padmanabhan and K. Srinivasan, *Hawking radiation in different coordinate settings: complex paths approach*, *Class. and Quant. Grav.* **19** (2002) 2671 [gr-qc/0010042].

[38] S.W. Hawking, *Black hole explosions*, *Nature* **248** (1974) 30.

[39] S.W. Hawking, *Particle creation by black holes*, *Commun. Math. Phys.* **43** (1975) 199.

[40] R. Casadio, B. Harms and Y. Leblanc, *Microfield dynamics of black holes*, *Phys. Rev.* **D 58** (1998) 044014 [gr-qc/9712017].

[41] R. Brandenberger and D. Easson, *Some observational consequences of black hole remnants*, MCGILL-02-37.

[42] P. Chen and R.J. Adler, *Black hole remnants and dark matter*, gr-qc/0205106.

[43] P. Hut and M.J. Rees, *Constraints on massive black holes as dark matter candidates*, IASSNS-AST-92-52.

[44] D.J. Hegyi, E.W. Kolb and K.A. Olive, *Black holes and local dark matter*, *Astrophys. J.* **300** (1986) 492.

[45] B.J. Carr, *Black holes, pregalactic stars, and the dark matter problem*, FERMILAB-CONF-85-86-A.

[46] J.I. Kapusta, *Nucleation rate for black holes*, *Phys. Rev.* **D 30** (1984) 831.

[47] S.B. Giddings, *Black holes and massive remnants*, *Phys. Rev.* **D 46** (1992) 1347 [hep-th/9203059].

[48] C.G. Callan Jr., S.B. Giddings, J.A. Harvey and A. Strominger, *Evanescent black holes*, *Phys. Rev.* **D 45** (1992) 1005 [hep-th/9111056].

[49] T. Banks, A. Dabholkar, M.R. Douglas and M. O'Loughlin, *Are horned particles the climax of Hawking evaporation?*, *Phys. Rev.* **D 45** (1992) 3607 [hep-th/9201061].

[50] S.B. Giddings and A. Strominger, *Dynamics of extremal black holes*, *Phys. Rev.* **D 46** (1992) 627 [hep-th/9202004].

[51] R. Brandenberger and D. Easson, work in progress.

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